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Lösungen weitere Themen (alter LP) — W
Kurvendiskussion: Musteraufgabe — 09

1.

$D = \mathbb{R}$

A: $\lim\limits_{x\to\pm\infty} f(x) = 0$:
Waagrechte Asymptote $y = 0$

S: $f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x)$:
Punktsymmetrie zum Ursprung

N: $f(x) = 0$: $x = 0$

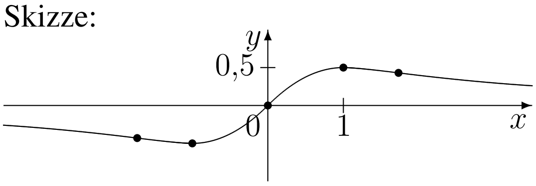
E: $f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$
$f'(x) = 0$: $1 - x^2 = 0$; $x_{1/2} = \pm 1$.

$f'<0$		$f'>0$		$f'<0$
fällt	-1	steigt	1	fällt
	Min		Max	
	$(-1\mid -\frac{1}{2})$		$(1\mid\frac{1}{2})$	

W: $f''(x) = \frac{(x^2+1)^2\cdot(-2x) - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{(x^2+1)\cdot(-2x) - (1-x^2)\cdot 4x}{(x^2+1)^3} = \frac{2x^3-6x}{(x^2+1)^3}$
$f''(x) = 0$: $2x^3 - 6x = 0$;
$2x(x^2-3) = 0$;
$x_1 = 0$, $x_{2/3} = \pm\sqrt{3}$.

$f''<0$		$f''>0$		$f''<0$		$f''>0$
re.-	$-\sqrt{3}$	li.-	0	re.-	$\sqrt{3}$	linksgekrümmt
	WP		WP		WP	
	$(-\sqrt{3}\mid -\frac{\sqrt{3}}{4})$		$(0\mid 0)$		$(\sqrt{3}\mid\frac{\sqrt{3}}{4})$	

Skizze:



$W_f = [-\frac{1}{2}; \frac{1}{2}]$

2.

Verschiedene Schreibweisen für den Funktionsterm:
$f(x) = \frac{1}{x} - x^2 = \frac{1-x^3}{x} = x^{-1} - x^2$

$D = \mathbb{R}\setminus\{0\}$

A: $\lim\limits_{x\to\pm\infty} f(x) \to -\infty$

$\lim\limits_{x\to\pm 0} f(x) = „\frac{+1}{\pm 0}“ \to \pm\infty$
(Senkr. As., Pol 1. Ordnung $x = 0$)

S: Keine Symmetrie

N: $f(x) = 0$: $1 - x^3 = 0$; $x = 1$

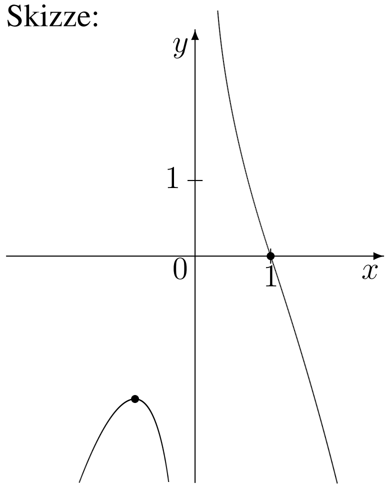
E: $f'(x) = -x^{-2} - 2x = \frac{-1-2x^3}{x^2}$
$f'(x) = 0$: $-1 - 2x^3 = 0$; $x = -\frac{1}{\sqrt[3]{2}}$

$f'>0$		$f'<0$		$f'<0$
steigt	$-\frac{1}{\sqrt[3]{2}}$	fällt	0	fällt
	Max		$\notin D$	
	$(-\frac{1}{\sqrt[3]{2}}\mid -\frac{3}{2}\sqrt[3]{2})$			

W: $f''(x) = +2x^{-3} - 2 = \frac{2}{x^3} - 2$
$f''(x) = 0$: $x^3 = 1$; $x = 1$

$f''<0$		$f''>0$		$f''>0$
re.-	0	li.-	1	rechtsgekrümmt
	$\notin D$		WP	
			$(1\mid 0)$	

Skizze:



$W_f = \mathbb{R}$